WORTHWHILE MATHEMATICS TASKS: A CONVERSATION TOOL

Task Features	Attributes The task… *Note: As tasks vary according to purpose and type, tasks will vary in attributes. The goal is to have a balance of tasks that as a collection develop all the mathematical proficiencies.*	Leadership Implications
Mathematical Focus *What is the mathematical learning that students will achieve with this task?*	• has a clear connection to the curriculum – for both content and processes • enabes students to develop a deeper understanding of mathematical concepts, skills and processes • broadens students' skills and mathematical proficiencies	
Connection to Prior Knowledge *How does the task build on students' prior knowledge and experience?*	• is developmentally connected to earlier mathematical concepts • addresses a clear student learning need within the zone of proximal development • may connect to known contexts or life experiences of the students • if contextual, the context is not overly complicated or distracting from the mathematics	
Problem Solving *Is the task problematic for students? (i.e. something they need to think about, not simply a disguised way of practising already demonstrated algorithms)*	• necessitates the use of a range of higher-level thinking skills in mathematical processes • allows students to develop, select and apply problem solving strategies • requires explanation, justification or proof, and thinking time • allows for developing and applying reasoning, hypothesis making and the testing of various strategies • provides students opportunities to reflect on and demonstrate that they are monitoring their thinking to help clarify their understanding	
High Cognitive Demand and Understanding *Does the task provide opportunities to "press for understanding"?*	• requires students to explore and understand the nature of mathematical concepts, processes, or relationships • has the potential to develop students' mathematical processes (see below) and/or their proficiencies (see overleaf) • provides appropriate level of challenge for every student, encouraging students to address their misconceptions • has an element of complexity and academic press, encouraging students to see mathematics as sense making • is visually structured so as not to obscure the inherent mathematical thinking and reasoning required	
Tools and Representations *Does the task allow for the use of multiple tools and representations?*	• allows students to select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems • promotes the creation of a variety of representations of mathematical ideas • encourages the communication of thinking either orally, visually, or in writing	
Entry Points *Does the task allow for multiple entry points for students?*	• is open, encouraging multiple approaches to its solution • has a low floor for entry and a high ceiling for exit; thus is accessible to a wide range of learners • provides opportunity for "mathematical play", i.e. involves investigation, conjecturing and problem extensions	
Intellectual and Social Engagement *Does the task have the potential to engage students in mathematical thinking?*	• has the potential to spark students' curiosity (there is an element of perplexity) • considers the use of concise questions, visuals, video and other approaches that appeal to the intended audience • allows for student choice and voice; students must make decisions about what to do and how to do it • can be solved in a reasonable amount of time • provides opportunities for students to have their thinking valued, share their thinking, inquire, build on the ideas of others and reconstruct or confirm their thinking	
Additional Input *What other features could be considered?*		
Task Type *Which type of task is this?*	Tasks can be classified as one of four types: purposeful representative, contextual, content specific, or practice/consolidation. How does this task contribute to a balance of mathematical tasks and thus one or more of the mathematical proficiencies?	

The **mathematical process expectations** in the Ontario Curriculum are embedded within the attributes above

Problem Solving Reasoning and Proving Reflecting Selecting Tools and Computational Strategies Connecting Representing Communicating

Mathematical Communities of Practice Conversation Tool
Creating a Non-Threatening Classroom Environment and Classroom Discourse
Adapted from Anthony & Walshaw (2007)

A non-threatening classroom atmosphere is one in which there is an acknowledgement that all students have the capacity to become powerful mathematics learners. Effective teachers develop a trusting climate and also a culture of high expectations. **Effective classroom discourse** involves supporting students in developing the mathematical language skills required for explanation, argumentation and justification of their mathematical ideas.

Mathematics teaching for diverse learners demands…

Ethic of Care	**Space for the Individual and the Collective**
• How do you promote relationship building? • How do you value students' contributions, including mistakes? • How do you nurture social relationships? • How do you build confidence in the students? • How do you show that you care about the development of mathematical proficiency? • How do you use culturally relevant and responsive pedagogical practices?	• What are the norms of mathematical participation? • Do the students sometimes work in groups? • What opportunities are there for students to work with and learn from peers? • Is there individual thinking time?
Explicit Instruction	**Respectful Exchange of Ideas**
• How are you shaping students' mathematical language? • How are you initiating student reflection on the concept? • How are you initiating student reflection on the process? • How are you eliciting information to determine how the students have interpreted the mathematical concept? • How are you connecting the underlying meaning of a mathematical concept in English with ELL students' home languages?	• Do students have opportunities to explain and justify their solutions? • What kind of constructive feedback is being provided to students? • Does the feedback engage the learners in further purposeful knowledge construction? • Is there time given for productive struggle before feedback is offered? • Are you repeating, rephrasing, or expanding student talk in order to clarify or highlight content, extend reasoning, include new ideas, or move the discussion in another direction? • Are you encouraging students to make connections among different discoveries? • Are you inviting multiple representations of ideas?
Teacher Content Knowledge, Knowledge of Mathematics Pedagogy, and Reflecting-in-Action	
• How are you making sense of the mathematics? • How are you building your mathematics content knowledge? • How are you building your mathematics pedagogical knowledge (knowledge of how to teach the content)? • How are you making connections between aspects of mathematical knowledge? • How are you delving into the minds of students by noticing and listening carefully to what students have to say? • How are your reflections about student learning informing your instructional decision-making?	

Effective mathematical pedagogy is a coherent system rather than a set of discrete, interchangeable strategies. The pedagogical system encompasses a non-threatening classroom environment, instructional tasks, tools and representations, and classroom discourse. These four aspects of effective pedagogy constitute a system, for example, the way instructional tasks are realized in the classroom and experienced by students depends on the classroom atmosphere, the tools and representations available for them to use and the nature and focus of classroom discourse.

The Mathematical Proficiencies
Adapted from *Adding it up: Helping children learn mathematics* (National Research Council, 2001

1. **Conceptual understanding** – comprehension of mathematical concepts, operations and relations
2. **Mathematical fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately, and in addition to these procedures, factual knowledge and concepts that come to mind readily
3. **Strategic competence** – ability to formulate, represent and solve mathematical problems
4. **Adaptive reasoning** – capacity for logical thought, reflection, explanation and justification
5. **Productive disposition** – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (growth mindset)

The Five Practices
5 Practices for Orchestrating Productive Mathematics Discussions (Smith & Stein, 2011)

After setting goals and selecting tasks, the teacher engages in five practices to prepare for instruction:

Anticipating what students will do and what strategies they will use in solving a problem correctly and incorrectly
Monitoring by paying close attention to students' mathematical thinking as they work on the proposed task, identifying the mathematical learning potential of particular strategies
Selecting students whose strategies are worth discussing in class to get certain ideas on the table while remaining in control of which students present and what the mathematical content of the discussion will likely be
Sequencing by making decisions about how to order the students' presentations to maximize the chances that the mathematical learning goals for task are achieved
Connecting the strategies and ideas in a way that build on each other to help students develop an understanding of the mathematical ideas

Guidelines for Whole-Class Math Talk
Adapted from *What Works? Research Into Practice* #1 (Bruce, 2007)

1. **Explain** "This is my solution/strategy…" "I think ____ is saying that…."
2. **Agree** with reason: "I agree [with that method] because…."
3. **Disagree** with reason: "I disagree [with that method] because…."
4. **Build** on: "I would like to build on that idea…."
5. **Go** beyond: "This makes me think about…" "Another way to think about this is…"
6. **Wait** time

Question Types that Elicit Student Thinking
Adapted from *What Works? Research Into Practice* #59 (Suurtamm, Quigley, Lazarus, 2015)

1. **Seeking** an Alternative Method: So you used one strategy here, is there another strategy you could use to represent how you could work on this problem?
2. **Posing** New Challenges: So what do you think we should do next?
3. **Promoting** Group Interaction: Okay, do you want to explain to ___ what you are doing?
4. **Encouraging** Sense Making: What does it make sense to you?